MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2017/2018

DIM5068 – MATHEMATICAL TECHNIQUES 2

(For DIT students only)

4^{th} JUNE 2018
9.00 a.m – 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **2** pages with **4** questions excluded the cover page and Appendix. Key formulae are given in the Appendix.
2. Answer **ALL** questions.
3. Write your answers in the answer booklet provided.
4. All necessary working steps must be shown.

Question 1

a. Differentiate the following functions with respect to x by using **Chain Rule**.

i) $f(x) = \cos(x^3 - 5x^2 - 4x)$. (5 marks)

ii) $y = 2\ln(3x^2 + 4)$ (6 marks)

b. Differentiate $x^3 y + 2y^2 - 7x^2 = x^2 y$ by using **implicit differentiation.** (7 marks)

c. Given $f(x) = 4x^3 + 6x^2 - 24x - 12$

i) Find the intervals on which the function is increasing and decreasing (5 marks)

ii) Identify the function's local extreme values (2 marks)

[TOTAL 25 MARKS]

Question 2

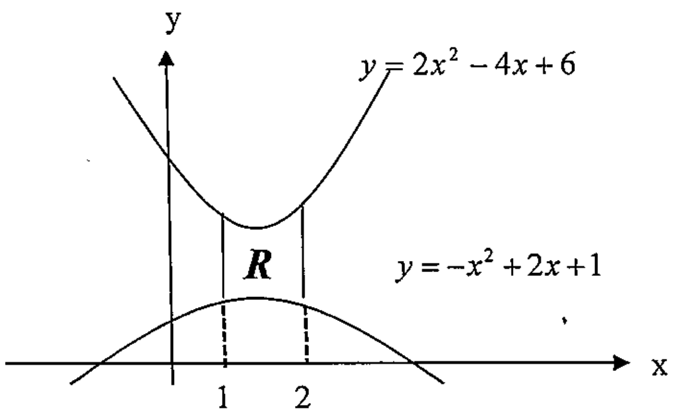
a. Find/evaluate the following integrals:

i) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (4x^2 + \sin x)dx$ (5 marks)

ii) $\displaystyle\int (4x^3)(2x^4 - 2)^6 \, dx$ [Hint: Use **Substitution Rule**] (5 marks)

iii) $\displaystyle\int_0^2 \frac{x}{\sqrt{x^2}} dx$ [Hint: Use **Substitution Rule**] (7 marks)

b. Find the area of the region $\boldsymbol{R}$ bounded by $y = 2x^2 - 4x + 6$, $y = -x^2 + 2x + 1$, $x = 1$ and $x = 2$. (8 marks)



[TOTAL 25 MARKS]

Continued...

Question 3

a. Solve the following differential equation by using the **Separable Method**.

i) $\sqrt{2+4x^3}\,dy = y^4x^2\,dx$. (Hint: **Solve for y**) (8 marks)

ii) $3\dfrac{dy}{dx} = 6x^2y + 3y$. (Hint: **No need to solve for y**) (4 marks)

b. Given the differential equation $4\dfrac{dy}{dx} + 8y = 4e^{-2x}$

i) Identify the $p(x)$ and $q(x)$. (3 marks)

ii) Calculate the integrating factor, μ. (1.5 marks)

iii) Find y given $\mu y = \int \mu q(x)dx$. (3.5marks)

[TOTAL 20 MARKS]

Question 4

a. Given $\vec{a} = \langle -3,2,1 \rangle$ and $\vec{b} = \langle 2,0,-2 \rangle$, find

i) $3\vec{a} + 4\vec{b}$ (3 marks)

ii) $|3\vec{a} + 4\vec{b}|$ (2 marks)

b. Find the angle between $\vec{u} = 4\vec{i} - 3\vec{j}$ and $\vec{v} = 2\vec{i} + 5\vec{j}$. (5 marks)

c. Determine whether $\vec{v} = 3\vec{i} - 2\vec{j}$ and $\vec{w} = 4\vec{i} + 6\vec{j}$ are parallel, orthogonal, or neither. (3 marks)

d. Given the vertices of a triangle $X = (0, -2, 0)$, $Y = (4, 1, -2)$, and $Z = (5, 3, 1)$.

i) Determine $\overrightarrow{XY}$ and $\overrightarrow{XZ}$. (2 marks)

ii) Calculate the cross product of $\overrightarrow{XY}$ and $\overrightarrow{XZ}$. (3 marks)

iii) Compute the total area of the triangle. Correct your answer to 2 decimal places. (3 marks)

e. If a line passing through the points (1, 3, 2) and (-4, 2, 0), compute the

i) parametric equations of the line. (4 marks)

ii) symmetric equations of the line. (3 marks)

f. Find an **equation of the plane** that passes through the point (1,4,-5) and is perpendicular to $3\vec{i} + 2\vec{j} - 2\vec{k}$ .Leave your answer in the form of $ax + by + cz = d$. (2 marks)

[TOTAL 30 MARKS]

End of page.

APPENDIX

Derivatives: $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Differentiation Rules

General Formulae

1. $\frac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$
2. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$
3. $\frac{d}{dx}(x^n)=nx^{n-1}$
4. $\frac{d}{dx}[f(u)]=\frac{dy}{du}.\frac{du}{dx}$

Exponential and Logarithmic Functions

1. $\frac{d}{dx}(e^x)=e^x$
2. $\frac{d}{dx}(a^x)=a^x\ln a$
3. $\frac{d}{dx}(\ln x)=\frac{1}{x}$
4. $\frac{d}{dx}(\log_a x)=\frac{1}{x\ln a}$

Trigonometric Functions

1. $\frac{d}{dx}(\sin x)=\cos x$
2. $\frac{d}{dx}(\cos x)=-\sin x$
3. $\frac{d}{dx}(\tan x)=\sec^2 x$
4. $\frac{d}{dx}(\csc x)=-\csc x\cot x$
5. $\frac{d}{dx}(\sec x)=\sec x\tan x$
6. $\frac{d}{dx}(\cot x)=-\csc^2 x$

Table of Integrals

1. $\int u\,dv=uv-\int v\,du$
2. $\int u^n\,du=\frac{u^{n+1}}{n+1}+C,\quad n\neq -1$
3. $\int\frac{du}{u}=\ln|u|+C$
4. $\int e^u du=e^u+C$
5. $\int\sin u\,du=-\cos u+C$
6. $\int\cos u\,du=\sin u+C$
7. $\int\sec^2 u\,du=\tan u+C$
8. $\int\csc^2 u\,du=-\cot u+C$
9. $\int\sec u\tan u\,du=\sec u+C$
10. $\int\csc u\cot u\,du=-\csc u+C$

Application of Integrals:

Areas between Curve, $A=\int_a^b[f(x)-g(x)]\,dx$

Differential Equations

Linear Differential Equations

$$\frac{dy}{dx} + p(x)y = q(x) \quad \Rightarrow \quad \mu y = \int \mu q(x)\,dx, \text{ where } \mu = e^{\int p(x)\,dx}$$

Constant Coefficient of Homogeneous Equations

Roots of Auxiliary Equation, $r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

General Solutions to the Auxiliary Equation:

2 Real & Unequal Roots ($b^2 - 4ac > 0$)	$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
Repeated Roots ($b^2 - 4ac = 0$)	$y = c_1 e^{rx} + c_2 x e^{rx}$
2 Complex Roots ($b^2 - 4ac < 0$)	$y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

Constant Coefficient of Non-Homogeneous Equations

$y = y_c + y_p$ [y_c: complementary solution, y_p: particular solution]

Vector

Length of Vector

The length of the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Dot Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}| \cos \theta$

Cross Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \times \mathbf{b} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta$

Area for parallelogram *PQRS*

$= \left|\vec{PQ} \times \vec{PR}\right|$

Area for triangle PQR

$= \dfrac{1}{2}\left|\vec{PQ} \times \vec{PR}\right|$

Equation of Lines

Vector equation: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$

Parametric equations: $x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$

Symmetric equation: $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

Equation of Planes

Vector equation: $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Linear equation: $ax + by + cz + d = 0$

Angle between Two Planes: $\theta = \cos^{-1}\left(\dfrac{\mathrm{n}_1 \bullet \mathrm{n}_2}{|\mathrm{n}_1||\mathrm{n}_2|}\right)$